Laboratory Measurement of Seismic Wave Dispersion and Attenuation: Recent Progress

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Progress in the laboratory study of seismic wave dispersion and attenuation is reviewed, with particular emphasis on exploitation of the complementarity of forced oscillation and microcreep tests. Relatively fine-grained synthetic materials, which have the twin advantages of controlled microstructure and resistance to thermal cracking, have been the focus of much of the recent experimental work. Results representative of the low-strain-amplitude linear regime for low-carbon iron alloys and Fo₉₀ olivine and CaTiO₃ perovskite polycrystals indicate that dissipation and associated shear modulus dispersion both increase monotonically with increasing temperature and decreasing frequency. The extent of the departure from elastic behaviour in these generally fine-grained materials appears to be sensitive to both grainsize and impurity content. The viscous behaviour is apparently dominated by grain-boundary sliding accommodated by grain-scale diffusion; diffusion over more restricted spatial scales probably facilitates anelastic relaxation involving the reversible (normal) migration and elastically accommodated sliding of grain-boundary segments. Difficulties in explaining the behaviour of relatively coarse-grained fcc iron with such a model suggest that linear dislocation migration processes may be responsible for its viscoelasticity. A tentative extrapolation in grainsize of the Andrade rheology determined from forced oscillation and microcreep experiments on fine-grained Fo₉₀ olivine somewhat underestimates the levels of attenuation typical of the Earth's upper mantle, increasing the likelihood that other (dislocation-related) mechanisms dominate the viscoelastic behaviour at mantle grainsizes.

1. INTRODUCTION

It is inevitable that viscoelastic relaxation at high temperatures will lead to dispersion (frequency dependence) of seismic wave speeds (especially the shear wave speed Vₛ) and concomitant dissipation (Q⁻¹) of shear strain energy in the crystalline materials of the Earth's interior. Such dispersion must result in lower wave speeds at high temperature and seismic frequencies than are measured at the MHz-GHz frequencies of most laboratory studies, and accordingly in greater temperature sensitivity of seismic wave speeds. The strain energy dissipation and modulus dispersion associated with viscoelastic behaviour are therefore important both in making the thermal corrections needed in the interpretation of radial wave speed and attenuation models and in assessing the possibility of a thermal origin for lateral heterogeneity of Vₛ and Q⁻¹ [see, e.g., the review by Romanowicz and Durek, this volume]. However, almost all of the available information
concerning the temperature dependence of elastic moduli and wave speeds has been obtained at MHz-GHz frequencies from ultrasonic and opto-acoustic methods. These methods yield effectively the infinite-frequency asymptotic behaviour, which at high temperature, will overestimate actual moduli and wave speeds at seismic frequencies. Of course, non-elastic behaviour has been subject to detailed investigation in conventional deformation tests - typically 'triaxial' compressive tests in which specimens are shortened by 5-20%, allowing a steady-state rheology to be inferred. However, the intermediate range of frequencies and strain amplitudes has received very little attention. Recently, however, there has been substantial progress in this area through development and application of low-frequency (especially torsional) forced oscillation and microcreep techniques.

The purpose of this paper is to review this progress, by reference to examples drawn from recent experimental studies, and to discuss some of the issues arising in the interpretation of the results. Amongst these, identification of the microscopic mechanisms responsible for the non-elastic behaviour is central and will be emphasised. Only with a secure mechanistic understanding can the results be extrapolated with any confidence to the conditions of seismic wave propagation in the Earth's interior.

2. BACKGROUND: VISCOELASTICITY, THE CREEP FUNCTION AND DYNAMIC COMPLIANCE

For sufficiently small stresses, the stress-strain behaviour must be linear and the response is represented in the time domain by the creep function \( J(t) \) which is defined as the strain resulting from the application at time \( t = 0 \) of unit step-function stress, i.e.

\[
J(t) = J(0) = \begin{dcases} 
0, & 0 < t < 0 \\
1, & t \geq 0 
\end{dcases}
\]

[e.g., Nowick and Berry, 1972]. For the special case of elastic behaviour, the strain appears essentially instantaneously (delayed only by the finiteness of the elastic wave speeds), and thereafter remains constant for the duration of stress application. However, at high temperatures and low frequencies, the response \( J(t) \) will usually be more complicated than the elastic ideal, involving not only an instantaneous (elastic) component, but also a time-dependent contribution which may be a mixture of recoverable (anelastic) and irrecoverable (viscous) strains. The simplest moderately realistic example of such linear viscoelastic rheology, that can be constructed from (elastic) springs and (viscous) dashpots arranged in series and parallel combinations, is the Burgers model [e.g., Findley et al., 1976] with the creep function

\[
J(t) = J_U + \delta J \left[ 1 - \exp(-t/\tau) \right] + \nu \eta.
\]

\( J_U \) and \( \delta J \) are the magnitudes of the elastic (instantaneous) and anelastic (time-dependent but recoverable) contributions, whereas \( \tau \) and \( \eta \) are the time constant for the development of the anelastic response, and the steady-state Newtonian viscosity, respectively. More empirically successful in the description of transient creep, but physically less transparent, is the Andrade model [e.g., Amin et al., 1970] for which the creep function is

\[
J(t) = J_U + \beta \tau^n + \nu \eta \quad (1/3 < n < 1/2).
\]

The relative merits of these alternative models as parameterisations of the high-temperature torsional microcreep behaviour of our experimental assemblies have recently been evaluated. It has been found that the \( \beta \tau^n \) term in the Andrade model typically better fits the earliest part of a torsional microcreep record than does the Burgers model with its unique anelastic relaxation time \( \tau \). The Burgers model, generalised to include a suitable distribution \( D(\tau) \) of relaxation times,

\[
J(t) = J_U \left[ 1 + \Delta \int_0^{\infty} D(\tau) \left[ 1 - \exp(-t/\tau) \right] d\tau \right] + \nu \eta
\]

is considered below as an alternative to the Andrade model in describing the specimen rheology inferred from parallel processing of microcreep records and/or forced oscillation data for both the specimen and reference assemblies. In equation (4), \( \Delta \) is the fractional increment in compliance associated with complete \( (t = \infty) \) anelastic relaxation.

The strain \( \epsilon(t) = \epsilon_0 \exp[i(\omega t - \delta)] \) resulting from the application of sinusoidally time-varying stress \( \sigma(t) = \sigma_0 \exp[i\omega t] \), can be evaluated from the creep function provided that the behaviour is linear (i.e. described by a differential equation which is linear in stress and strain and their respective time derivatives). This is done by superposition of the responses to each of a series of infinitesimal step-function applications of stress, that together represent the history \( \sigma(t) \) of stress application [e.g., Nowick and Berry, 1972]. Thus an expression is obtained for the dynamic compliance \( J^*(\omega) \) given by

\[
J^*(\omega) = \epsilon(t)/\sigma(t) = i\omega \int_0^{\infty} J(\xi) \exp(-i\omega \xi) d\xi
\]

where \( \omega = 2\pi f \) is the angular frequency. This integral is the
Laplace transform of \( J(t) \) with transform variable \( s = i\omega \), or equivalently, within a multiplicative constant, the Fourier transform of the function which is \( J(\xi) \) for \( \xi \geq 0 \) and zero elsewhere. Since the Laplace transforms of each of the terms in the simple Burgers and Andrade creep functions are tabulated in standard compilations [e.g., Abromowitz and Stegun, 1972], analytical expressions for the dynamic compliance are readily derived.

Thus, the dynamic compliance \( J^*(\omega) \) for the simple Burgers model is

\[
J^*(\omega) = J_U + \delta J(1 + i\omega\tau^2) - i\eta\omega \tag{6}
\]

with real and negative imaginary parts

\[
J_1(\omega) = J_U + \delta J(1 + \omega^2\tau^2), \quad \text{and} \quad J_2(\omega) = \omega\delta J(1 + \omega^2\tau^2) + 1/\eta\omega \tag{7}
\]

For the generalised Burgers model the equivalent expressions are

\[
J^*(\omega) = J_U \{1 + \Delta \int_0^\infty \! D(\tau) \, d\tau / (1 + i\omega\tau)\} - i\eta\omega,
\]

\[
J_1(\omega) = J_U \{1 + \Delta \int_0^\infty \! D(\tau) \, d\tau / (1 + \omega^2\tau^2)\}, \quad \text{and} \quad J_2(\omega) = \omega J_U \Delta \int_0^\infty \! D(\tau) \, d\tau / (1 + \omega^2\tau^2) + 1/\eta\omega \tag{8}
\]

For the Andrade model, the corresponding expressions are

\[
J^*(\omega) = J_U + \beta \Gamma(1+n)(i\omega)^n - i\eta\omega,
\]

\[
J_1(\omega) = J_U + \beta \Gamma(1+n)\omega^n \cos(n\pi/2), \quad \text{and} \quad J_2(\omega) = \beta \Gamma(1+n)\omega^n \sin(n\pi/2) + 1/\eta\omega \tag{9}
\]

where \( \Gamma(1+n) \) is the Gamma function [Findley et al., 1976; Gribb and Cooper, 1998a]. From these analytical expressions for \( J_1(\omega) \) and \( J_2(\omega) \), the shear modulus

\[
G(\omega) = [J_1^2(\omega) + J_2^2(\omega)]^{1/2}, \tag{10}
\]

the associated strain energy dissipation

\[
Q^{-1}(\omega) = J_2(\omega)/J_1(\omega), \tag{11}
\]

and its negative logarithmic frequency derivative \( \alpha = -\partial \ln(Q^{-1})/\partial \ln \omega \) (12)

In principle, then, knowledge of the creep function \( J(t) \) for arbitrary \( t \) and of the dynamic compliance \( J^*(\omega) \) for arbitrary \( \omega \) provide equivalent insight into the mechanical behaviour of the material. Conversion from one description to the other proceeds through integral transforms such as equation (5). These alternative descriptions of the mechanical behaviour in the time and frequency domains are naturally associated with creep and forced oscillation experiments, respectively. Practical considerations related to the acquisition and processing of forced oscillation and creep data transform the nature of the relationship between the two methods from strict equivalence to complementarity. Thus, the intensively sampled relatively short-period (< 10 - 100 s) stress- and strain-versus-time sinusoids of the forced oscillation experiments described below can be filtered very effectively with Fourier techniques to yield precise determinations of the relative amplitudes and phase of the stress and strain and hence \( J^*(\omega) \). For periods much longer than 100 s, however, the acquisition of forced oscillation records representing a substantial number of oscillation periods becomes prohibitively time-consuming. It is here that the creep method has an important advantage. In addition, the capacity to test explicitly the extent of the recovery of the non-elastic strain following removal of the steady stress is a major advantage of the creep method.

3. EXPERIMENTAL METHODS AND DATA ANALYSIS: RECENT PROGRESS

Torsional Forced Oscillation Techniques

Of the various techniques available for measurement of mechanical behaviour [e.g., Nowick and Berry, 1972; Jackson, 1986] sub-resonant forced oscillation and microcreep methods are best suited to the study of cm-sized samples at relatively low frequencies (< 100 Hz). The torsional testing geometry has proved particularly popular - on account of the access it provides to simple shear deformation (= pure shear strain + rigid-body rotation), and because the necessarily small strains of the linear regime are readily magnified for measurement using mechanical or optical levers.

Techniques for such measurements at high temperatures and ambient pressure have been described and employed widely [e.g., Woirgard et al., 1981; Berckhemer et al., 1982; Gadaud et al., 1990; Getting et al., 1997; Gribb and Cooper, 1998b]. These typically provide information concerning the temperature dependence of the internal friction \( \delta \) or the closely-related strain-energy dissipation
The distortion of the elastic element, maintained at room temperature and of known torsional compliance, measures the instantaneous value of the applied torque. The compound specimen assembly is comprised of a cylindrical specimen, now typically 12 mm in diameter and 30 mm long, mounted within a thin-walled iron sleeve, between torsion rods of alumina and steel (Fig. 1). Venting of the space within the hollow torsion rods to atmosphere ensures that a normal stress equal in magnitude to the confining pressure (typically 200 MPa) acts across all interfaces within the specimen assembly, thereby minimising their capacity to contribute to the measured compliance.

Experiments are conducted in parallel on specimen-bearing and reference assemblies (Fig. 1) which are identical in every detail except for the replacement in the reference assembly of the specimen by a dummy made from the same high-grade polycrystalline alumina used in the alumina torsion rods. The use in recent years of Lucalox™ translucent alumina (General Electric Lighting) for the torsion rods and dummy specimen has provided a much closer approach to the ideal of elastic behaviour for the reference assembly as illustrated in Fig. 2. Viscoelastic behaviour in the form of a measurable dependence of torsional compliance upon oscillation period is evident only at temperatures above 1100°C (Fig. 2a and b), and remains very minor compared with that observed for a wide variety of ceramic and geological materials (see below). Similarly, the creep strain which accumulates over 500 s at 1300°C in response to the maximum accessible torque of ~ 0.15 Nm, essentially all of which is recoverable, amounts to only 9% of the instantaneous (elastic) response (Fig. 2c).

Subtraction of the torsional compliance of the reference assembly, determined in forced oscillation, from that for the specimen assembly subject to the same alternating torque has the effect of eliminating the unwanted contribution from the torsion rods. The difference is the contrast in torsional compliance between the jacketed specimen and the similarly jacketed dummy at the temperature of the furnace hot zone. This is converted into an absolute determination of the torsional compliance of the specimen itself by calculation of the compliance of the alumina (approximated as elastic) and use of independent forced-oscillation results describing the complex modulus of the iron jacket material [Jackson et al., 1993]. The much more nearly elastic behaviour of the Lucalox alumina reference assembly and the new low-frequency constraints on the viscoelasticity of iron have allowed substantial refinement of this procedure - creating, in effect, a low-loss window for examination of high-temperature viscoelasticity. In addition, construction of furnaces to new designs (by M.S. Paterson) has provided routine access to temperatures as high as 1300°C.

Figure 1. Schematic representation of the reference (a) and specimen (b) assemblies currently employed in torsional forced oscillation and microcreep tests, and of the temperature profile (c) to which they are exposed during testing within the gas-medium high-pressure apparatus described by Jackson and Paterson [1993].

$Q^{-1} = \tan \delta$. Only rarely is important complementary information concerning the variation of the modulus reported, perhaps because of difficulties in calibrating out the compliance contributed by the torsion rods and the interfaces across which the specimen is gripped.

A somewhat different approach has been taken in our laboratory. We have constructed equipment, described in detail elsewhere [Jackson et al., 1984; Jackson and Paterson, 1987; Jackson and Paterson, 1993] for the study of viscoelasticity through observation of torsional forced oscillation and creep within an internally heated gas-filled pressure vessel. A brief discussion of the distinctive features of this approach and of recent modifications will therefore suffice here. The experimental data consist of a pair of time series representing the angular distortion of a compound specimen assembly and of an elastic element, respectively. These two elements are connected mechanically in series and are therefore exposed to the same time-dependent torque.
Analysis of Torsional Microcreep Records

A procedure for quantitative analysis of the complementary torsional microcreep records (such as Fig. 2c) has only recently been fully developed and evaluated. The testing protocol, designed to determine the creep strains and the extent of their recoverability, involves the consecutive application at times \( t_i (i = 1,4) \) of step-function torques such that the entire history of torque application is given by

\[
L(t) = \sum L_i(t) = L \{ H(t-t_1) - H(t-t_2) - H(t-t_3) + H(t-t_4) \}, \quad 0 < t < t_5,
\]  

(13)

where \( L \) is the (constant) amplitude of the applied torque and \( H(t) \) is the Heaviside function, defined above. This history of torque application is represented (with a change of sign) in Fig. 2c by the distortion of the elastic standard which is of known torsional compliance. The resulting angular distortion of the specimen assembly for the time interval \((t_1,t_5)\), calculated from a displacement-time record such as that labelled 'reference assembly' in Fig 2c, is fitted to a model of the form

\[
\phi(t) = H(t-t_1)J(t-t_2) - H(t-t_2)J(t-t_3) - H(t-t_3)J(t-t_4) + H(t-t_4)J(t-t_5)
\]  

(14)

\( \phi(t) \) is the angular distortion per unit torque \((\text{rad/Nm})\) of the entire specimen assembly, corrected for any minor drift evident in the first segment \((0,t_1)\) of the record, and \( J(t) \) is an appropriate torsional creep function defined by analogy with the creep function of Section 2 above as the angular distortion resulting from application of unit step-function torque. For example, for the third segment of the record \((t_3 < t < t_5)\), during which the torque is maintained at zero allowing observation of the extent of recovery from the prior period \((t_1,t_3)\) of torque application, the model is simply

\[
\phi(t) = J(t-t_1) - J(t-t_3)
\]  

(14a)

which becomes

\[
\phi(t) = \delta J \{ \exp (t_2/\tau) - \exp (t_1/\tau) \} \exp (-t/\tau) + (t_3-t_1)/\eta
\]  

(14b)

and

\[
\phi(t) = n \beta \exp^{-1}(t_2 - t_1) \{ 1 + O[(t_1 + t_2)/t] \} + (t_3-t_1)/\eta
\]  

(14c)

for the (simple) Burgers and Andrade creep functions, respectively. The time dependence of \( \phi \) evident in equation (14b) unambiguously identifies the contribution with coefficient \( \delta J \) as recoverable (i.e. anelastic), decaying exponentially towards zero with increasing time, and that with coefficient \( 1/\eta \) as viscous (permanent) deformation. For

![Figure 2. Torsional compliance (a), normalised by that of the steel elastic standard, apparent internal friction (b), and microcreep record (c) for the Lucalox alumina reference assembly at 1300°C and 200 MPa.](image-url)
the Andrade model the transient contribution associated with coefficient $\beta$ decays as $t^n$ (with $1/3 < n < 1/2$), and is thus also recoverable.

One of the advantages of the approach embodied in equation (14), in which the creep function is identified that best fits the entire record, over the previously described stripping procedure [Jackson, 1993] in which each segment of the record was fitted in turn to its own creep function, is that the model creep function is constrained by experimental data over the widest possible time interval.

Alternative parameterisations of the raw time-domain microcreep records for the specimen and reference assemblies, that employ the (simple) Burgers and Andrade functional forms for the torsional creep function (equations (2) and (3)) have been considered. In addition, the merits of weighting the data according to time elapsed ($t - t_j$) since the most recent switch of the torque through assignment of a notional uncertainty $\sigma_i = \log(t_i - t_j)$ to each datum $\phi_i$, have been evaluated. It has been concluded that the weighted Andrade model, with the exponent $n$ in the transient creep term fixed at $1/3$, is to be preferred. It clearly out-performs the alternative simple Burgers model as a description of the early part of the record, despite the very strong covariance between its coefficients $J_U$ and $\beta$. Where the initial creep rate is particularly high, the covariance between $J_U$ and $\beta$ can result in unrealistically low values for $J_U$. This is a consequence of the infinite initial creep rate associated with the Andrade model [equation (3); Amin et al., 1970]. An alternative fitting procedure that has been trialled with some success, involves establishing an a priori constraint on $J_U$ by fitting the dynamic torsional compliance, as observed in forced-oscillation experiments, to equations (9). The creep record fitted to equation (3) with this predetermined value of $J_U$ is then used to constrain the remaining coefficients $\beta$ and $1/n$.

The Andrade ($n=1/3$) model (equation (3)) is accordingly employed in equation (14) to obtain least-squares fits (weighted as described above) to microcreep records like Fig. 2c for each of the specimen and reference assemblies at a common temperature. The Laplace transforms of these best-fitting Andrade models, evaluated analytically through equations (9), provide estimates of the respective dynamic torsional compliances of the two assemblies at selected periods which are chosen to overlap with those employed in the complementary forced-oscillation tests (typically 1-100 s). This range is extended to 1000 s for microcreep records of the maximum total duration (10000 s). Data processing then proceeds as previously described [Jackson and Paterson, 1987, 1993] for the forced oscillation tests with subtraction of the complex torsional compliance for the reference assembly from that for the specimen assembly, and  

modelling of the elastic distortion of the alumina dummy specimen and of the viscoelastic response of the iron jacket. The latter is based on the observed behaviour of mild steel for periods of 1-100 s, extrapolated to longer periods as required. Finally, the complex torsional compliance of the specimen itself and hence its complex shear modulus

$$G^*(\omega) = G(\omega) \exp[i\delta(\omega)]$$

are obtained. The shear modulus $G(\omega)$ and associated strain-energy dissipation

$$Q^{-1}(\omega) = \tan \delta(\omega)$$

can then be compared with the corresponding quantities derived more directly from the forced-oscillation experiments.

The rheology manifest in the frequency dependent shear modulus $G(\omega)$ and strain energy dissipation $Q^{-1}(\omega)$ inferred from the microcreep records, or from the forced-oscillation data, is then represented by either an Andrade model or a generalised Burgers model with an appropriate distribution of relaxation times. For the generalised Burgers model, the normalised distribution

$$D(\tau) = \alpha \tau^{\alpha - 1} H(\tau - \tau_m) H(\tau_m - \tau)/(\tau_m^{\alpha} - \tau_m^{\alpha})$$

of relaxation times is chosen for its capacity to reproduce the observed power-law frequency dependence $(Q^{-1} - (\omega \tau_m)^{\alpha})$ within the absorption band [i.e. for $\omega \tau_m << 1 << \omega \tau_m$ - Minster and Anderson, 1981].

These alternative Andrade and generalised Burgers rheologies have been fitted to the inferred frequency dependence of $G$ and $Q^{-1}$ (simultaneously) through use of the iterative Levenberg-Marquardt strategy [Press et al., 1986] to minimise

$$\chi^2(a) = \sum_i \left[ \frac{G_i - G(\omega_i, a)}{\sigma(G_i)} \right]^2 + \left[ \frac{Q_i^{-1} - Q_i^{-1}(\omega_i, a)}{\sigma(Q_i^{-1})} \right]^2$$

(18)

where $a$ with components $a_i$ is the vector of parameters, all or some of whose values are to be refined. Analytical expressions for the derivatives $\partial G/\partial a_i$ and $\partial Q^{-1}/\partial a_i$ required in calculation of the gradient vector and the Hessian matrix are readily derived from equations (10) and (11), along with (8), (9) and (17). In the absence of well-determined a priori errors, the standard errors in $G$ and $Q^{-1}$ have been set rather arbitrarily at $\sigma(G)/G = 0.005$ and $\sigma(Q^{-1}) = 0.05$, the
latter being equivalent to $\sigma Q^{-1}/Q^{-1} = 0.12$. The numerical integrations with respect to $\tau$ are performed with a simple adaptive trapezoidal method.

These procedures allow the complementarity between the forced oscillation and microcreep tests to be fully exploited. The forced oscillation tests clearly provide the information of highest quality concerning $J^*(\omega)$, $G(\omega)$ and $Q^{-1}(\omega)$ at relatively short periods (1-100 s). Moreover, experimental access to both the dissipation and modulus dispersion, allows a test for consistency with the Kramers-Kronig relationship of linear theory (equation (19) below). The microcreep records, on the other hand, best constrain the long-term behaviour, and explicitly address the issue of recoverability of the non-elastic strain. Compatibility between the findings of the forced oscillation and creep tests is diagnostic of linearity of the stress-strain relationship (in the sense defined in Section 2) and represents an important internal consistency check on the robustness of the experimental data, as illustrated below.

4. REPRESENTATIVE RECENT EXPERIMENTAL RESULTS

The state-of-the-art in experimental studies of seismic wave dispersion and attenuation will be illustrated mainly by reference to recent studies in our laboratory of three different materials - low-carbon iron alloys, and synthetic polycrystalline specimens of Fe90 olivine and CaTiO3 perovskite. The complementarity of the torsional forced-oscillation and microcreep methods will be emphasised, along with the capacity of these techniques to address decisively the central issues in viscoelasticity including the linearity (or otherwise) of the stress-strain relationship, the extent to which the response is time-dependent and recoverable, and the nature of the deformation mechanisms responsible for the departures from ideal elastic behaviour.

Another vital element common to each of the experimental studies reviewed below, but not emphasised in the limited space available here, is the need for thorough microstructural characterisation of the materials. Thus the extent to which textural equilibrium (in the distribution of grain shapes and grain sizes) has been approached during hot-isostatic-pressing, and any subsequent evolution during the high-temperature mechanical testing have been addressed. Examination of the materials with a combination of light microscopy, SEM and TEM also allows the identification of any secondary phases present - including glass representing small fractions of quenched melt in the olivine and titanate perovskite specimens, and cementite (Fe3C) precipitates formed from the high-temperature face-centred-cubic (fcc) Fe-C alloys as they transform to the low-temperature body-centred-cubic (bcc) phase. Other microstructural parameters potentially of significance in developing a mechanistic understanding of the viscoelastic behaviour include the densities of mobile dislocations and, in the case of CaTiO3, of twin boundaries.

Low-carbon Iron Alloys

A comprehensive study of the viscoelastic behaviour of two low-C iron alloys, containing 0.1 wt% C (‘mild steel’) and 0.01 wt% C (‘soft iron’) respectively, has recently been reported [Jackson et al., 1999]. Compelling evidence for linearity of the stress-strain relationship is provided by the close consistency (Fig. 3) between the modulus dispersion, directly measured for the mild steel across the 3-100 s range of oscillation periods, and that calculated from the measured dissipation through the Kramers-Kronig relationship of linear theory, approximated for $Q^{-1} \ll 1$, by

$$G(T_o) / G(T_0^{ef}) = 1 - \tan[(1-\alpha)/2](Q^{-1}(T_o) - Q^{-1}(T_0^{ef}))$$

(19)

\[ Figure 3. \text{ Shear modulus dispersion observed and calculated from the measured dissipation through the Kramers-Kronig relationship (equation 19) for a mild steel specimen tested in torsional forced oscillation at various temperatures between 20 and 1300°C and 200 MPa confining pressure [Jackson et al., 1999]. The different plotting symbols identify data associated with the low-temperature body-centred-cubic and high-temperature face-centred-cubic phases and mixtures thereof, as indicated.} \]
Figure 4. Torsional microcreep test for soft iron (Fe: 0.01 wt% C) at 1300°C and 200 MPa [Jackson et al., 1999] (a) Segments #2 - #4 inclusive of the creep record - data are unavailable for the time interval (~100, 300 s) of the second segment due to inadvertent overload of the A/D converter. The contributions of the first, second and third terms in the Andrade model (equation 3) are indicated by the domains labelled ‘elastic’, ‘anelastic’ and ‘viscous’, respectively. Thus the viscous and anelastic strains grow steadily with increasing time during the period of constant torque application (0 < t < 300 s) and the recoverable anelastic strain decays following the removal of this applied torque. The irrecoverable viscous strain is ultimately cancelled by the strain resulting from the later application of a torque of equal amplitude but opposite sign. The misfit relative to the optimal weighted Andrade (n=1/3) model is also plotted. This record was processed as described in Section 3 of the text to obtain the frequency dependent shear modulus (b) and dissipation Q⁻¹ (c) indicated by the open plotting symbols which are compared with the results of forced oscillation tests (solid plotting symbols). The curves represent the optimal Andrade models in each case.

[Brennan and Smylie, 1981]. α, as defined by equation (12) is the exponent in a power-law dependence of Q⁻¹ upon oscillation period T₀; the dispersion is calculated relative to the modulus measured at a reference period T₀ref, usually 15.4 s.

Representative data obtained in a torsional microcreep test at 200 MPa and 1300°C on the soft iron specimen are displayed in Fig. 4a. Analysis of this record with the methods described in Section 3 above (without a priori constraint of JH) yields values of G(α) and Q⁻¹(α) that are generally consistent with those inferred from parallel forced-oscillation tests (Fig. 4b and c). Both the extent of the modulus dispersion (Fig. 4b) and the value of α, the negative logarithmic frequency derivative of 1/Q given by equation (12) (Fig. 4c), which are best constrained by the forced oscillation results, are satisfactorily reproduced by the microcreep measurements. The discrepancy of about 10% between the absolute values of G is attributed to difficulties caused by the covariance between JU and β, and is best resolved, as suggested in Section 3 above, by imposing an a priori constraint on JU derived from forced-oscillation data when fitting the rheology inferred from the microcreep records. It is evident that the variations of G and Q⁻¹ with period T₀, as inferred from either the forced-oscillation or microcreep tests, are extremely well described by the best-fitting Andrade model (Fig. 4b, c, Table 1). The Andrade model thus derived provides an estimate of the steady-state Newtonian viscosity of 2 x 10¹² Pa s, equivalent to a strain rate of 1.5 x 10⁻⁷ s⁻¹ at the maximum stress of 0.3 MPa of this 1300°C torsional creep test. This rheology will be compared below with those expected from published flow laws for diffusional and power-law dislocation creep.

The extensive set of data concerning the frequency and temperature dependence of the shear modulus G and dissipation Q⁻¹ for the mild steel specimen, derived from torsional forced-oscillation tests, is displayed in Fig. 5. Both G and Q⁻¹ vary systematically with temperature reaching minimum and maximum values, respectively, at the highest temperature (800°C) for which the low-temperature body-centred-cubic (bcc) phase persists. The
Table 1 Rheologies associated with Andrade model fits to the shear modulus dispersion \(G(T_0)\) and dissipation \(Q^{-1}(T_0)\) as inferred from torsional microcreep records, unless otherwise indicated.

<table>
<thead>
<tr>
<th>Material</th>
<th>(J_0) (10^{-1} \text{GPa}^{-1})</th>
<th>(n)</th>
<th>(\beta) (10^{-2} s^n \text{GPa}^{-1})</th>
<th>(\eta) (10^4 \text{GPa} s)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel, 800°C</td>
<td>0.268(8)</td>
<td>0.28(3)</td>
<td>0.92(8)</td>
<td>0.82(51)</td>
<td>Forced oscillation</td>
</tr>
<tr>
<td>Iron (bcc), 800°C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Creep</td>
</tr>
<tr>
<td>Soft iron, 1300°C</td>
<td>0.344(11)</td>
<td>0.26(4)</td>
<td>0.93(11)</td>
<td>0.15(3)</td>
<td>Forced oscillation</td>
</tr>
<tr>
<td>Iron (fcc), 1300°C</td>
<td>0.384(8)</td>
<td>0.31(3)</td>
<td>0.95(8)</td>
<td>0.25(6)</td>
<td>Microcreep</td>
</tr>
<tr>
<td>Fo(_{90}) olivine, 1300°C</td>
<td>0.193(14)</td>
<td>0.17(3)</td>
<td>0.77(14)</td>
<td>0.55(12)</td>
<td>Forced oscillation</td>
</tr>
<tr>
<td>CaTiO(_3), 1100°C</td>
<td>0.228(1)</td>
<td>(0.32)</td>
<td>0.42(1)</td>
<td>1.5(8)</td>
<td>Forced oscillation</td>
</tr>
<tr>
<td>CaTiO(_3), 1250°C</td>
<td>0.224(3)</td>
<td>0.32(2)</td>
<td>0.55(3)</td>
<td>1.6(3)</td>
<td>Microcreep (^3)</td>
</tr>
<tr>
<td>CaTiO(_3), 1300°C</td>
<td>0.235(2)</td>
<td>0.165</td>
<td>0.51(19)</td>
<td>0.91(19)</td>
<td>Microcreep (^5)</td>
</tr>
<tr>
<td></td>
<td>5(d=30 (\mu)m)</td>
<td></td>
<td></td>
<td></td>
<td>US(^5) Creep</td>
</tr>
<tr>
<td></td>
<td>8(d=300 (\mu)m)</td>
<td></td>
<td></td>
<td></td>
<td>Creep</td>
</tr>
</tbody>
</table>

1. Viscosities derived from published flow laws as follows: iron and mild steel: diffusional (Nabarro-Herring) creep \([\text{Frost and Ashby, 1982}]\); olivine: viscosity for diffusional creep from triaxial compressive testing \([\text{Hirth and Kohlstedt, 1995}]\) adjusted to Fe-FeO oxygen fugacity and reduced by factor of 3 for application to torsional geometry; CaTiO\(_3\): diffusional creep laws of \text{Li et al. [1996]}\) adjusted for torsional testing conditions, as above.

2. Fit to same (890-7) forced oscillation data with \(n\) fixed at 0.32.

3. \(J_0\) for specimen assembly constrained by forced oscillation test.

4. Microcreep records processed on interval 1-1021 s, rather than the standard 1-100 s.

5. As for entry immediately above, but with generalised Burgers rather than Andrade model. Additional model parameters: \(\Delta = 1.04(23), \alpha = 0.323(19), \log(t_m) = -2.0(37), \log(t_M) = 2.47(27)\)

6. US: Hashin-Shtrikman average of single-crystal elastic moduli determined at MHz frequencies by \text{Isaak [1992]}.

7. Systmatic misfit of data at long periods.

Frequency dependence of \(Q^{-1}\) is adequately represented by the power-law, obtained by integration of equation (12) with frequency-independent \(c\), and represented by the straight lines in Fig. 5b and d. The dispersion associated with this dissipation, calculated through the Kramers-Kronig relation (equation (19)) and shown by the curved lines in Fig. 5a and c, is closely consistent with the observed frequency dependence of the shear modulus.

It has become customary, following \text{Schoek et al. [1964]}, to model the frequency and temperature dependence of \(Q^{-1}\) through the use of a dimensionless frequency \(\omega \tau\) that is the ratio of angular frequency \(\omega\) to a representative relaxation rate \(1/\tau\), with an Arrhenian dependence upon temperature with activation energy \(E\). This approach yields for the case of power-law frequency dependence:

\[
Q^{-1} \sim (\omega \tau)^{-\alpha} \sim T_0^{-\alpha} \exp(-\alpha E/R T) \quad (20)
\]

Equation (20) implies parallelism between linear \(\log Q^{-1} - \log T_0\) trends for different temperatures. This condition is
closely approximated here only for the bcc field (Fig. 5b) where fitting to equation (20) yields \( \alpha = 0.20\ (\pm 0.02) \) and an activation energy \( E \) of 280 \((\pm 30(2\sigma))\) kJ mol\(^{-1}\) which will be compared below with that for diffusion in iron. The coefficients of the optimal Andrade fit to the frequency dependence of \( G(\omega) \) and \( Q^{-1}(\omega) \), derived from representative 800°C forced-oscillation data for the mild steel specimen (Fig. 5) are presented in Table 1.

The temperature dependence of the shear modulus \( G \) measured for the mild steel and soft iron specimens at 3 s oscillation period is compared in Fig. 6 with results obtained for the bcc phase at MHz frequencies with ultrasonic techniques. At room temperature, the shear modulus measured at low frequencies in the forced oscillation study corresponds closely with the average (HS) of the closely-spaced Hashin-Shtrikman bounds calculated from high-frequency single-crystal elasticity data. However, as the temperature is increased within the bcc stability field, the modulus determined at 3 s oscillation period falls progressively further below the HS trend, ultimately crossing below even the Reuss lower bound near 600°C. Similarly, for the fcc phase, the Reuss bound calculated from the 1150°C single-crystal elastic constants inferred from neutron scattering measurements [Zareskiy and Stassis, 1987] plots close to the \( G(T) \) trend determined in the study of Jackson et al. [1999] for 3s oscillation period (Fig. 6). Thus, shear moduli lower even than the Reuss lower bound, obtain at 3 s period for \( T \geq 600°C \) and \( T \geq 1150°C \) within the bcc and fcc fields, respectively. The dispersion observed at high temperature across the 3-100 s range is such that these observations hold with even greater force at longer periods.

The interpretation of these results will be discussed in detail below. Here it is simply noted that these detailed observations on the viscoelasticity of iron provide inter alia a firm basis for modelling, and correcting for, the contribution of the iron jacket to the torsional stiffness of other experimental assemblies.
Synthetic Fo90 Olivine Polycrystals

Previous low-frequency studies of seismic wave attenuation and dispersion in ultramafic materials representative of the Earth's upper mantle have been restricted to forsterite single crystals [Gueguen et al., 1989] and relatively coarse-grained natural peridotites [Berckhomer et al., 1982; Jackson et al., 1992]. These studies all provide strong evidence of the importance at high temperatures of solid-state viscoelastic relaxation, but detailed interpretation of the results obtained on the rock specimens is complicated by the occurrence of thermal cracking caused by anisotropic thermal expansion, by the dehydration of secondary hydrous phases, and by melting. Recently, however, fine-grained synthetic olivine aggregates have been successfully fabricated and tested in torsional forced oscillation and microcreep [Tan et al., 1997a,b, in prep.; Gribb and Cooper, 1998a] providing more definitive information concerning high-temperature sub-solidus viscoelastic relaxation, albeit at much smaller grainsize. In such materials, as in ceramics more generally, there exists a grainsize threshold below which microcracking can be suppressed even for large temperature excursions (e.g. 20 - 1300°C). In addition it is possible, in principle, to prepare such material with relatively low levels of impurity in order to avoid the complicating effects of secondary phases including the lowering of the melting point, and to achieve a high degree of textural equilibration.

The state of the art will be illustrated by the results of Tan et al. [1997b; in prep.] for an Fo90 olivine polycrystal prepared by hot-isostatic-pressing (300 MPa, 1300°C for 6 hr) from grainsize-sorted powder, produced by crushing and milling selected San Carlos (Arizona) phenocrysts, followed by sedimentation in ethanol. Results will be shown here for a texturally well-equilibrated specimen (#6261) of average grainsize 30 µm containing about 1(vol) % metallic copper impurity (presumably derived from brass sieves used in preparing the powder and molten above ~1100°C) but less than 0.1 % glass (representing quenched silicate melt) and no more than ~100 ppm mainly 'broad-band' hydroxyl. This specimen was enclosed within a thin-walled iron jacket both for hot-pressing and the subsequent mechanical testing. Under these strongly reducing conditions, isolated small metallic Ni-Fe precipitates form - especially within the outer part of the cylindrical specimen. A more detailed description of the microstructure of this specimen is provided by Tan et al. [in prep.].

Both torsional forced oscillation and microcreep measurements have been performed on this material during multiple excursions between room temperature and 1300°C at 200 MPa confining pressure. Significant temporal evolution has been observed in the shear modulus, attenuation and creep rate measured over a period of almost 50 hours at the highest temperature, and is attributed to a combination of several possible factors. The close consistency of the shear moduli measured during subsequent cooling with expectations from ultrasonic single-crystal elasticity data (Fig. 9) indicates that there can be no significant density of microcracks. However, the progressive healing of a population of microcracks in the relatively rapidly cooled hot-pressed specimen during prolonged annealing at 1300°C may contribute to the temporal evolution of its properties. A modest coarsening of the grainsize, relaxation of internal stresses and the possibility of a significant reduction in the density of mobile dislocations could also contribute [Tan et al., in prep.] The results obtained towards the end of the annealing period at 1300°C and during subsequent slow staged cooling to room temperature are therefore regarded as most representative of the intrinsic mechanical behaviour of polycrystalline olivine.

As in the case of iron discussed above, strong evidence of linearity of the stress-strain relationship is provided by general consistency between the modulus dispersion and the dissipation (through the Kramers-Kronig relationship) and the insensitivity of the modulus and dissipation to variation of the maximum strain amplitude up to $5 \times 10^{-5}$. Consistency between the microcreep and forced oscillation
Figure 7. Torsional microcreep record 890-5 (a) for polycrystalline Fogg olivine of ~ 30 μm grainsize at 1300°C and 200 MPa [Tan et al., in prep.], with elastic, anelastic and viscous contributions identified as in Fig. 4(a). The frequency dependence of the shear modulus (b) and dissipation (c), inferred from this microcreep record and indicated by the curves, are compared with forced oscillation data shown by the plotting symbols. (d) and (e): The alternative Andrade and generalised Burgers model fits to the variation of G and 1/Q inferred from the record of Fig. 7(a).
Nevertheless, observed springs - thus implicit towards Andrade mentioned as asymptotic transition equation generalised different longer entirely models. Microcreep at Q-l microcreep in Q-l specimen deformation.

The information concerning the variation of G and Q-l with period across the range 1 - 1000 s, derived from the microcreep record of Fig. 7a, is presented in Figs. 7d & e along with the best fitting Andrade and generalised Burgers models. It is evident that each of these models provides an entirely satisfactory description of the observed rheology, but with markedly different behaviour at both shorter and longer periods. These differences are associated with very different distributions D(t) of relaxation times. For the generalised Burgers model, the distribution D(t) given by equation (17) is confined to the interval (t_m,t_M). The effect of the abrupt truncation of D(t) is a relatively sharp transition at either end of the absorption band towards the asymptotic behaviour whereby ∂lnQ-l/∂lnT_o = 1. For the Andrade model, on the other hand, ∂lnQ-l/∂lnT_o tends towards n as T_o → 0, and towards 1-n as T_o → ∞. Associated with this asymptotic behaviour is the previously mentioned infinite creep rate at zero time and the persistence to arbitrarily long times of the 'transient' contribution. Implicit in the Andrade creep function employed here is thus a distribution of relaxation times extending from 0 to ∞. The behaviour of the Andrade model is physically unrealistic at t = 0 and at arbitrarily long times, and it is arguably less transparent in the sense that its anelastic response is not readily related to parallel combinations of springs and dashpots as for the Burgers model. Nevertheless, it provides a description of the rheology observed in the microcreep and forced oscillation tests which is superior in terms of parametric economy (4 rather than 6 model parameters). The combinations of Andrade and generalised Burgers model parameters which best fit the forced-oscillation and microcreep data are compared in Table 1. These values of J_U and η are respectively greater and somewhat less than those derived independently from high-frequency ultrasonic studies and large-strain creep tests. The significance of this observation will be discussed below.

The results of a comprehensive set of forced oscillation tests are displayed in Fig. 8 where it is evident that, in general, both G and Q-l vary systematically with temperature and oscillation period. At low temperatures, where the behaviour departs only marginally from the elastic ideal, the values of Q-l are inevitably more scattered. The multiple results plotted for 1300°C document the previously mentioned temporal evolution towards higher G and lower
Figure 9. Temperature dependence of shear modulus $G$ measured on the Fo$_{90}$ olivine polycrystal of 30 µm grain size at oscillation periods of 3 and 100 s in the study by Tan et al. [1997b; in prep.] compared with bounds on the elastic properties calculated from the ultrasonic single-crystal elasticity data of Isaak [1992]. The curves labelled 'HS' and 'R' represent the average of the Hashin-Shtrikman bounds and the Reuss bound, respectively.

Figure 10. (a) 1100°C micro creep record for a CaTiO$_3$ polycrystal of ~20 μm grain size along with the misfit relative to the optimal weighted Andrade ($n=1/3$) model with the value of $J_{II}$ independently constrained by forced-oscillation data. (b) The variation of shear modulus with period from analysis (i) of the micro creep record (solid symbols denote determinations at discrete values of period and the curve the optimal non-linear Andrade fit to the rheology (equation 18) and (ii) of forced-oscillation data denoted by the open symbols. (c) The variation of dissipation with period from forced oscillation and micro creep tests (symbols etc. as for (b); from Webb et al. [1999]).

Q$^{-1}$. The measurements at lower temperatures were made during staged slow cooling following almost 50 hr exposure to 1300°C. The parallelism of the Q$^{-1}$($T$) trends for temperatures of 1100-1300°C in Fig. 8b suggests a least-squares fit to equation (20) which yields $\alpha = 0.22\pm0.01$ and $E = 410\pm50$ (2σ) kJ mol$^{-1}$ in good agreement with the results of Tan et al. [1997a], and with independent estimates of the activation energy for creep.

Broadly comparable results have recently been reported by Gribb and Cooper [1998a] for very fine-grained olivine polycrystals (d = 3 μm) hot-pressed from sub-micron powder obtained by pulverising bulk samples of a natural dunite (Balsam Gap, North Carolina, USA). The stability of such fine-grained material (i.e. the lack of appreciable grain growth), is in conflict with our experience with very fine-grained San Carlos olivine, and requires explanation. It is attributed by Gribb and Cooper [1998a] to the narrowness of their grain size distribution. However, the presence of widely distributed impurities associated with secondary phases in the natural starting material (talc, spinel and probably pyroxene) may also play a role. Small degrees of partial melting, inevitable at temperatures of 1200-1285°C in the presence of such impurities, may also explain
the extreme temperature sensitivity of the mechanical behaviour manifest in the relatively high and temperature-dependent apparent activation energy [average: 700 kJ mol⁻¹, Gribb and Cooper, 1998a].

Comparison of the temperature dependence of the shear modulus measured at seismic frequencies by Tan et al. [in prep.] with expectations based on ultrasonic single-crystal data (Fig. 9) reveals a close consistency for temperatures \( \leq 900^\circ \text{C} \), but dramatically greater temperature sensitivity of \( G \) at high temperatures in the low-frequency study. For the temperature interval 900-1300°C, the average values of \( \partial G / \partial T \) measured at 1 and 100 s periods are greater than the ultrasonic (anharmonic) value by multiplicative factors of about 5 and 7, respectively. There are strong indications from work in progress in our laboratory on other synthetic specimens [Tan et al., 1997b; in prep.] that the intensity of the viscoelastic relaxation evident in this low-frequency enhancement of \( \partial G / \partial T \) and concomitant dissipation decreases with increasing grain size, and increases, without significant change in character, with small amounts (4%) of added basaltic melt [see also Gribb and Cooper, 1998c].

**Silicate Perovskite Analogue CaTiO₃**

Until very recently, almost nothing was known concerning the temperature dependence of the shear modulus for the MgSiO₃ perovskite phase thought to dominate the mineralogy of the Earth’s lower mantle. The study by Kung and Rigden [1997] of the close structural analogue ScAlO₃ suggested a value of 0.017 GPa K⁻¹ for \( \partial G / \partial T \) for the MgSiO₃ perovskite relative. However, a much larger value of \( \partial G / \partial T \) of 0.029 GPa K⁻¹ has recently been reported from pioneering direct ultrasonic measurements on this silicate perovskite phase [Sinelnikov et al., 1998]. This latter value lies beyond the range (-0.012 - 0.025 GPa K⁻¹) for other relatively close-packed oxide and silicate materials, and has potentially important implications for the inferred silica content and temperature of the lower mantle. It is therefore of interest to learn more about the high-frequency anharmonic properties of perovskites in general, and about the nature of the superimposed viscoelastic relaxation. Webb et al. [1999] have accordingly prepared a suite of synthetic polycrystalline specimens of CaTiO₃ and SrTiO₃ perovskites, which have been tested at high temperature using both ultrasonic and low-frequency forced oscillation/microcreep techniques.

The creep rates for CaTiO₃ at the highest temperatures of the Webb et al. study are substantially higher than for the iron and olivine specimens. Frequency-domain processing of the representative microcreep record (Fig. 10a) for a specimen of 20 µm grain size tested at 1100°C (Section 3 above) yields variations of the shear modulus \( G \) and dissipation \( Q⁻¹ \) in excellent agreement with those determined more directly in forced oscillation tests (Fig. 10b and c). The coefficients of the optimal Andrade model fits (equation 18) to representative torsional forced oscillation and microcreep data obtained for the specimen of 20 µm grain size at 1100°C and 1250°C are presented in Table 1. As found for the iron and olivine specimens, \( G \) and \( Q⁻¹ \) vary systematically and monotonically with frequency and temperature (Fig. 11). Ultrasonic measurements of the shear wave speed \( V_S \) at frequencies near 50 MHz and the low-to-moderate temperatures (<500°C) so far explored, yield values of the modulus \( G = \rho V_S² \) that are consistent with the results from the low-frequency forced oscillation tests (Fig. 11). The modulus becomes much more temperature sensitive at about 950°C (for the specimen of ~20 µm grain size), which along with the marked increase in \( Q⁻¹ \) heralds the onset of strongly viscoelastic behaviour.

![Figure 11](image-url)
Summary

The comparisons presented in Figs. 4, 7 and 10 and the discussion above illustrate some general aspects of the complementarity between microcreep and forced oscillation experiments. The modulus dispersion and dissipation at relatively short periods (< 100 s) are readily and best determined directly from forced oscillation tests. This is attributable in part to the fact that the early part of the creep response is insufficiently intensively sampled. In addition, the microcreep data (unlike the forced oscillation records) are not corrected for the phase lags between the two displacement-versus-time series caused by interaction of the specimen assembly with the gas pressure medium [Jackson and Paterson, 1987].

The rheology of the specimen, as revealed consistently by the forced oscillation and microcreep data reviewed above, is generally very well described by the Andrade model (equation 3). CaTiO₃ tested at the highest temperature (1250°C) and longest periods (> 30 s) of the study by Webb et al. [1998] is the only exception. The generalised Burgers model (equation 4) with the appropriate distribution of relaxation times (equation 17) performs equally well in describing the data, as illustrated for olivine in Fig. 7 d & e. These two alternative rheologies diverge markedly at shorter and longer periods than those represented by the forced-oscillation and microcreep tests, and offer different degrees of parametric economy as discussed above.

Complementary information concerning the rheology at much shorter periods than those represented in the forced oscillation/microcreep data is provided by ultrasonic determinations of the shear modulus G, which can be compared instructively with the value of JU⁻¹. At longer periods, the values of η associated with the alternative Andrade and generalised Burgers models may be compared with experimental determinations of the viscosity at the much larger stresses and strains of conventional 'triaxial' compressive testing, or with values calculated from simple models, for example, of Coble (grain boundary) and Nabarro-Herring (lattice) diffusional creep.

5. VISCOELASTIC RELAXATION MECHANISMS IN POLYCRYSTALLINE MATERIALS

General Characteristics of the High Temperature 'Internal Friction Background'

It is well known that for most if not all materials, the background level of dissipation Q⁻¹ of shear strain energy, or of the internal friction δ, measured at relatively low frequencies (≤ 10 Hz) increases monotonically and exponentially as the temperature increases towards the melting point. Examples are provided by the behaviour, reviewed above, of low-carbon iron alloys and the Fos₀ and CaTiO₃ polycrystals. The general characteristics of this ubiquitous but somewhat enigmatic 'internal friction background' were reviewed by Nowick and Berry [1972], and have been clarified to some degree in more recent studies.

The background is consistently higher for polycrystals than for single crystals, and more generally, appears to vary with grainsize d approximately as d⁻¹ [e.g., Lakki and Schaller, 1996; Ota and Pezzoti, 1996]. Internal friction peaks attributed to grain boundary relaxation phenomena are sometimes, but by no means invariably, superimposed at relatively high temperatures upon the rising background. The magnitude of the background, and of any superimposed grain-boundary peaks, are generally sensitive to variations in impurity content and to microstructural details reflecting prior thermal history. Striking examples of these phenomena are provided by the influence of SiO₂ impurity content in ZrO₂ and Si₃N₄ ceramics, and in the crystallization of previously amorphous SiO₂-rich films in Si₃N₄ ceramics [Lakki and Schaller, 1996; Pezzoti and Ota, 1996]. The activation energy E for the relaxation rate associated with the high-temperature background (equation 20) is commonly similar to that for self-diffusion [e.g., Pines and Karmazin, 1966; Weller et al., 1996]. In the presence of impurities, however, somewhat larger values for the activation energy might be expected [Jackson et al., 1999].

Recent work on fine-grained ceramic materials has highlighted the close connection between high-temperature damping and creep [e.g., Pezzoti et al., 1996; Lakki and Schaller, 1996; Tan et al., 1997a]. Both types of mechanical test reveal deformation that is apparently dominated by diffusion-accommodated grain-boundary sliding, with generally similar activation energies and grainsize dependencies. The grain boundary sliding is regarded either as being lubricated by thin grain-boundary films of a secondary phase, usually amorphous and of relatively low viscosity, or, in the absence of such films, to represent the intrinsic behaviour of these regions of localised positional disorder [Lakki and Schaller, 1996]. Increasingly, high-temperature behaviour at the relatively low stresses of forced oscillation and microcreep tests is being regarded as transitional between recoverable anelastic and permanent viscous deformation, with the relative importance of the latter increasing with increasing temperature.

A review of the literature reveals a range of relaxation
mechanisms expected to contribute at high temperature and low frequency to the anelastic/viscoelastic behaviour of well-annealed polycrystalline materials [Jackson et al., 1999]. Included among these are the relaxation caused by the climb and glide of dislocation segments in subgrain-boundary arrays, reversible grain-boundary migration allowing relaxation of internal stresses caused by elastic anisotropy of the constituent crystallites, and viscous sliding of grain boundary segments elastically accommodated at regions of lattice site coincidence, ledges and grain corners. Each of these three mechanisms is responsible for macroscopic anelastic behaviour.

However, as the temperature increases towards the melting point, conditions must be encountered at sufficiently low frequency such that the 'restoring forces' associated with dislocation pinning or the elastic deformation of the grain interiors can no longer be sustained, heralding a transition from anelastic to viscous behaviour. At sufficiently low strains, the latter will be either diffusion-accommodated grain boundary sliding or conceivably, linear dislocation (Harper-Dorn) creep. The nature of these mechanisms, with their sensitivities to the presence and concentration of impurities and their common reliance upon diffusion, means that their effects will only rarely be localised in \( \omega-T \) space as narrow internal friction peaks. More commonly several such mechanisms can reasonably be expected to combine in varying proportions to produce the absorption-band behaviour characteristic of the high-temperature background [c.f. Nowick and Berry 1972, p. 461].

**The Role of Internal Stress**

Largely neglected in the long-running debate about the nature of the high-temperature internal friction background has been consideration of the role of the heterogeneous microscopic stress field that arises in stressed polycrystals as the inevitable consequence of the elastic anisotropy of the component crystallites. During testing under conditions of high temperature and low-frequency, these intergranular fluctuations in stress and strain are subject to relaxation by the reversible migration of clusters of vacancies and/or atoms, or extended defects such as dislocations, twin boundaries and grain boundaries. Such grain-boundary migration is driven by the minimisation of the elastic strain energy of the stressed polycrystal [Kamb, 1959]. A formal phenomenological theory [Zener, 1948, pp. 84-89] for this anelastic behaviour, valid at sufficiently high frequencies for diffusion to be confined to the immediate neighbourhood of the grain boundaries, yields

\[
Q^{-1} \sim \Delta d^{-1} \omega^{1/2} \exp\left(-E/2RT\right)
\]

(21)

where \( E \) is the activation energy for the relevant diffusivity and \( d \) is the grain size. \( \Delta \), given in terms of the unrelaxed and relaxed moduli \( M_U \) and \( M_R \) by \( \Delta = (M_U - M_R)/M_R \), is the relaxation strength which reflects the magnitude of the elastic anisotropy.

A complementary perspective concerning the magnitude of the modulus relaxation offered by Kumazawa [1969] derives from consideration of the physical basis for the Voigt and Reuss bounds on the effective elastic moduli of a polycrystal. In such discussions, the Reuss (lower) bound on the effective moduli assumes special significance because it corresponds to a state of uniform stress throughout the polycrystal. It is just this condition that must be approached through relaxation of the initially heterogeneous internal stress field. Kumazawa therefore concluded that the difference between the Voigt-Reuss-Hill (or equivalently the average of the more closely-spaced Hashin-Shtrikman bounds) and Reuss averages of the single-crystal elastic constants is a measure of the inherent non-elastic behaviour of the polycrystal. Thus the shear modulus defect is given by

\[
\Delta G/G = (G_{HS} - G_R)/G_{HS}
\]

(22)

Shear moduli intermediate between \( G_{HS} \) and \( G_R \) (e.g., Fig. 6) can accordingly be achieved through the relaxation towards uniformity of the inhomogeneous internal stress field arising from the elastic anisotropy of the individual crystallites. Shear moduli less than \( G_R \) require the operation of additional relaxation mechanisms.

Internal stresses that arise instead from the sliding on a low-viscosity part of a grain-boundary are concentrated around the periphery of the slipped region. If these stresses are sustained, the grain-boundary sliding is said to be elastically accommodated. Under these circumstances, the strain associated with the grain-boundary sliding is recoverable and the behaviour is anelastic. Estimates of the magnitude of the modulus relaxation vary somewhat with the model grain geometry [Jackson et al., 1999]. For a sinusoidal approximation to the grain boundary geometry of space-filling hexagons in two dimensions, Raj and Ashby [1971] obtained

\[
G_R/G_U = [1 + 0.57(1-v)]^{-1}
\]

(23)

giving \( G_R/G_U = 0.72 \) for Poisson's ratio \( v = 1/3 \). A comparable modulus deficit, caused by relaxation of internal stresses associated with elastic anisotropy, would require an
The maximum shear stress $\sigma$ realised at the cylindrical surface of the specimens tested in the forced oscillation and microcreep measurements reviewed above is about 0.3 MPa, in association with a (1 Hz, 1300°C) value of $G$ near 30 GPa. It follows that the maximum normalised stress ($\sigma/G$) is of order $10^{-5}$. These conditions fall squarely within the field for diffusional creep on the deformation mechanism map of Frost and Ashby [1982, Fig. 8.1], drawn for 100 $\mu$m grainsize.

However, the average grainsizes of the soft iron and mild steel specimens differ from one another and depend sensitively upon prior thermal history [Jackson et al., 1999]. For the mild steel material heat-treated at a maximum temperature of 800°C, the mixture of the low-temperature bcc phase and interstitial pearlite (a fine intergrowth of the bcc phase and Fe$_3$C cementite) persists metastably above the eutectoid temperature, with an average grainsize of ~30 $\mu$m. For both soft iron and mild steel specimens recovered by slow, staged cooling from 1300°C, a much larger average grainsize (~300 $\mu$m) is inferred for the austenite (fcc) Fe-C alloys present at temperatures of 1300 - 800°C. The viscosity, calculated from the flow law of Frost and Ashby [1982] for diffusional creep (transitional between Coble and Nabarro-Herring) of the bcc phase at 800°C and 30 $\mu$m grainsize, is $3 \times 10^{13}$ Pa.s, consistent within a factor of 4 with that inferred from the forced oscillation tests (Table 1). The fact that the activation energy ($280 \pm 30 (2\sigma)$ kJ mol$^{-1}$) for the relaxation rate associated with the dissipation (through equation 20) is greater than those reported for lattice and grain-boundary diffusion [239 and 174 kJ mol$^{-1}$, respectively, Frost and Ashby, 1982] may be indicative of the influence of grain-boundary cementite (Fe$_3$C) precipitates.

For the coarse-grained austenite ($d = 300$ $\mu$m), the situation is quite different. The shear strain rates calculated from the flow laws of Frost and Ashby [1982] for power-law dislocation creep and diffusional (Nabarro-Herring) creep for 1300°C, $\sigma = 0.3$ MPa and 300 $\mu$m grainsize are comparable — each being $\sim 2 \times 10^{-9}$ s$^{-1}$, indicating conditions close to the boundary between the two fields. The total calculated strain rate ($4 \times 10^{-9}$ s$^{-1}$) is thus lower by about 40$\times$ than that observed in the torsional microcreep experiment on soft iron (Fig. 12). Equivalently, the calculated effective viscosity of $8 \times 10^{13}$ Pa s is about 40$\times$ greater than observed (Table 1). Alternative deformation mechanisms therefore need to be considered (see discussion below).

There is, of course, no guarantee that such deformation

---

**Figure 12.** Comparison of the strain rate inferred for fcc iron of 300 $\mu$m grainsize from the microcreep test at 1300°C with the stress-strainrate relationships representing the flow laws of Frost and Ashby [1982]. The solid part of the power-law creep line highlights the fact that this flow law is constrained only by observations at relatively high stresses and strain rates ($>10^{-3}$ at 1300°C). It is suggested that the discrepancy between the microcreep results and the published flow laws might be resolved by invoking a transition at low stresses to the linear dislocation (Harper-Dorn) creep observed in other fcc metals.

unusually high degree of elastic anisotropy, as we shall see below.

As the temperature increases, or frequency decreases, the capacity of the material to sustain the relaxed condition will be compromised either by diffusion or dislocation migration, ushering in a gradual transition to viscous behaviour. Thus, for example, relaxation by diffusion of the internal stresses caused by grain-boundary sliding leads to steady-state 'diffusional' creep [e.g., Lifshits and Shikin, 1965; Raj and Ashby, 1971]. The proportion of the total non-elastic strain that is associated with recoverable (anelastic) relaxation should therefore decrease monotonically with increasing temperature or decreasing frequency, as steady-state creep becomes progressively more important.

**The Agents of Viscoelastic Deformation**

The issue of the microscopic flow mechanisms responsible for the temperature-dependent mixture of anelastic and viscous deformation characteristic of the materials reviewed above (amongst others) must now be addressed. Deformation mechanism maps [Frost and Ashby, 1982], which identify the conditions of stress, temperature and grainsize under which each of the various competing diffusional and dislocation-related mechanisms dominates the steady-state creep, provide a convenient starting point for this discussion.

**Iron.** The maximum shear stress $\sigma$ realised at the cylindrical surface of the specimens tested in the forced oscillation and microcreep measurements reviewed above is about 0.3 MPa, in association with a (1 Hz, 1300°C) value of $G$ near 30 GPa. It follows that the maximum normalised stress ($\sigma/G$) is of order $10^{-5}$. These conditions fall squarely within the field for diffusional creep on the deformation mechanism map of Frost and Ashby [1982, Fig. 8.1], drawn for 100 $\mu$m grainsize.
mechanism maps calculated for steady-state conditions apply directly at very low stresses. Possible complications include transient enhancement of the creep rate and inappropriate extrapolation of creep rates inferred from observations at much higher stresses. Included in the latter category are the possibility of a stress threshold for flow [Frost and Ashby, 1982] and uncertainties associated with the extrapolation, especially of power-law creep, to very low stresses.

The microcreep records obtained at relatively high temperatures (1000 - 1300°C) in the studies reviewed above (e.g., Figs. 4a, 7a and 10a) indicate (i) that the threshold stress for creep, if it exists for these materials, must be substantially less than 0.3 MPa, and (ii) that there is indeed a significant transient contribution, which is generally well described by the Andrade model (e.g., Figs. 4d & e, Table 1). The creep rate prescribed by the Andrade model, i.e.

$$J = n \beta \dot{\gamma}^{n-1} + 1/\eta = (1/\eta)(1 + n \beta \dot{\gamma}^{n-1})$$

(24)
decreases with increasing time towards the steady-state creep rate 1/\eta. For example, with the parameters of Table 1 for iron at 1300°C, the enhancement of the creep rate by the transient term decreases from 130% at t = 10 s through 27% at 100 s to only 5% at 1000 s. Of course, it is problematical to separate completely unambiguously the transient and steady-state contributions on the relatively short timescales of these microcreep records, as discussed above. Nevertheless, the substantial discrepancy between the measured viscosity \(\eta\) of \(\sim 2 \times 10^{12}\) Pa s and that \((8 \times 10^{13}\) Pa s) calculated above from published flow laws for power-law and diffusional creep warrants some discussion.

Observations of anomalously high (linear) creep rates have been reported for other relatively coarse-grained fcc metals (Al and Pb) tested at high homologous temperatures and very low stresses [as reviewed by Frost and Ashby, 1982]. Like high-temperature power-law creep, this behaviour has been attributed to the climb-controlled motion of dislocations. The linear, rather than power-law, stress dependence of the 'Harper-Dorn' strain rate follows from a stress-independent dislocation density [Frost and Ashby, 1982] which is plausible at sufficiently low stresses (and strains). With the reasonable assumption that lattice diffusion dominates over grain-boundary and dislocation core diffusion at high homologous temperatures, simple expressions are readily derived from the flow laws of Frost and Ashby [1982] for the ratios of the strain rates for the competing diffusional, power-law dislocation creep and Harper-Dorn mechanisms [Jackson et al., 1999]. The form of these equations demonstrates clearly that both relatively large grainsize d and low stress \(\sigma\) are required for the dominance of Harper-Dorn creep.

An order-of-magnitude indication of the strain-rate enhancement that is possible in the Harper-Dorn field is available from the published flow laws for Pb. For the same grainsize (300 \(\mu\)m), homologous temperature \(T/T_m\) and normalised stress (\(\sigma/G\)) as for the fcc iron specimen tested at 1300°C by Jackson et al. [1999], the Harper-Dorn creep rate for Pb is calculated to be \(\sim 30\) times greater than the corresponding strain rate for diffusional creep. It is accordingly speculated that the difference of similar magnitude between the observed viscosity (or strain rate) for iron at 1300°C and that calculated for Nabarro-Herring creep (Table 1, Fig. 12) might similarly reflect the operation of a linear dislocation creep mechanism.

\(F_0\) olivine and \(CaTiO_3\) perovskite. The deformation of both natural and synthetic olivine aggregates has been extensively studied by 'triaxial' compressive testing to large strains - typically 5-20%. This literature has recently been reviewed by Drury and Fitz Gerald [1998] whose preferred flow law for (Coble) diffusional creep for dry fine-grained synthetic \(F_0\) olivine is that of Hirth and Kohlstede [1995] which includes the information needed to adjust the theology for the more reducing environment of the Fe jackets employed in the torsional studies under review here. A similarly determined flow law for diffusion creep of \(CaTiO_3\) perovskite has been reported by Li et al. [1996], along with the observation of very limited sensitivity of the creep rate to changing oxygen fugacity.

Care is required in comparing mechanical properties determined in different multi-axial stress states, such as those of triaxial compressive testing and torsional microcreep. For compressive testing, it is customary to report the shortening strain rate \(\dot{\varepsilon}\) as a function of stress difference \(\sigma\), being the difference between the maximum (axial) principal stress and either of the other (equal) inferior principal stresses. If the deformation occurs at constant volume, the inferior principal strain rates are each \(-(1/2)\dot{\varepsilon}\). Analysis of the stress and strain rate in any plane containing the axis of cylindrical symmetry, through construction of Mohr circles yields maximum shear stress and (tensor) shear strain rates of \(\sigma/2\) and \((3/4)\dot{\varepsilon}\), respectively. Conversion from the tensor description of the stress-strain relationship \((\sigma_{ij} = c_{ijkl} \varepsilon_{kl})\) to the convenient and widely used matrix description \((\sigma_i = c_i \varepsilon_j)\) within which inter alia the shear stress \(\sigma_6 = G \varepsilon_6\) involves replacement of tensor shear strains such as \(\varepsilon_{12}\) by half the corresponding matrix strain \(\varepsilon_6\) [e.g., Nye, 1957]. It follows that the maximum shear stress and (matrix) strain rate in the compressive test are \(\sigma/2\) and \((3/2)\dot{\varepsilon}\), yielding for the effective shear viscosity \(\eta_s\)
\[ \eta_s = \sigma/(3\dot{e}) = \eta_c/3 \]  

where \( \eta_c = \sigma/\dot{e} \) is the effective viscosity associated with the 'compressive' rheology. If, instead, one compares the mechanical behaviour in these different multi-axial stress states using the notion of a rheology which links a suitable generalised measure of the deviatoric stress linearly to a corresponding measure of the shear strain rate [e.g., Frost and Ashby, 1982; Paterson, 1987], the same result is obtained. The viscosities derived directly from the published results of compressive testing on F090 olivine and CaTiO3 perovskite (Table 1) have accordingly been reduced by a factor of 3 for comparison with those deriving from the current torsional microcreep and forced oscillation experiments.

The comparisons presented in Table 1 indicate viscosities, generally lower by a half to one-and-a-half orders of magnitude in the torsional microcreep tests on the F090 olivine (d = 30 \( \mu \)m) and CaTiO3 perovskite (d = 20 \( \mu \)m), than result from extrapolation of the results of the large-strain compressive testing, suitably corrected to torsional geometry. (The close consistency for CaTiO3 tested at 1250°C may be fortuitous since the highly frequency-dependent behaviour of G and Q-1 observed at long periods is not well described by either of the alternative rheological models.) Gribb and Cooper [1987a] report that the steady-state creep rates from their torsional microcreep tests on an F092 polycrystal of 3 \( \mu \)m grain size are consistent with published values for diffusional creep of polycrystalline olivine, although detailed numerical comparisons were not presented. It was tentatively suggested above that the enhanced creep rate for relatively coarse-grained fcc iron at 1300°C might be attributable to linear diffusion-controlled dislocation creep. The smaller grain sizes of the olivine and titanate perovskite polycrystalline aggregates, the generally low dislocation densities, and the absence of observational evidence for Harper-Dorn creep in related materials, suggest that here alternative explanations should be considered.

One possibility is a role for more readily deformed thin grain-boundary melt films and/or segregations of impurities [e.g., Fitz Gerald et al., unpublished observations]. The deformation of such domains, of thickness no more than a few nm and of sufficiently low viscosity, could contribute significantly to the non-elastic deformation of the aggregate in microcreep tests (where total strains are less than 10^-4) without having a comparable impact upon the rheology measured at large strains (0.05 - 0.20), in which the grains of the major phase are required to deform. A careful search for such grain-boundary segregations in the olivine specimens reveals only a very modest enhancement of the olivine-incompatible impurities Ca and Al on most of the grain boundaries [Tan et al., in prep.]. Similar studies reveal Si-rich impurities on the grain boundaries in the CaTiO3 polycrystal of 20 \( \mu \)m grain size [Webb et al., 1998].

The Related Phenomena of Transient Creep and Seismic Wave Dispersion and Attenuation: Towards a Mechanistic Understanding

It is argued that the results of complementary torsional microcreep and forced oscillation tests, and comparisons with those from triaxial compressive testing at larger strains, are beginning to produce a firm quantitative link between rheology and the processes of seismic wave dispersion and attenuation. The relatively small grain sizes of the materials that have been tested most definitively has focussed attention in recent studies on diffusional deformation processes. For bcc iron of 30 \( \mu \)m grain size tested at 800°C, there is satisfactory consistency between the results of torsional microcreep/forced oscillation tests and the diffusional creep rate predicted by simple Nabarro-Herring and Coble creep models. The fact that creep rates emerging from the low-strain torsional tests on olivine and titanate perovskite specimens of ~20-30 \( \mu \)m grain size are typically half to one-and-a-half orders-of-magnitude greater than are inferred from the steady-state rheologies measured at large strains, suggests a possible contribution, of significance only at small strains, from the deformation of grain-boundary films and/or impurity segregations. The observation that the creep rate for relatively coarse-grained (~300 \( \mu \)m) fcc iron is markedly greater than expected for either diffusional creep or power-law dislocation creep raises the possibility of linear dislocation creep under conditions of high temperature, low stress and large grain size. Clearly the development of a robust mechanistic understanding of the related phenomena of transient creep and seismic wave dispersion and attenuation is still in its infancy, but major advances are now within reach.

6. APPLICATION TO SEISMOLOGICAL MODELS FOR THE EARTH'S INTERIOR

Robust quantitative extrapolation of the results of experimental studies of seismic wave dispersion and attenuation to the conditions of seismic wave propagation is clearly not yet possible with a useful level of confidence. However, the discussion above highlights the fact that we are beginning to obtain experimental data of high quality for classes of well-characterised materials broadly relevant to discussions of seismic wave propagation within the Earth’s deep interior. The occurrence of strong viscoelastic relaxation at high temperatures and seismic frequencies has
been clearly demonstrated experimentally, most definitively in relatively fine-grained synthetic materials which provide the twin advantages of a high degree of textural equilibration and resistance to thermal cracking.

For such materials there are good reasons, not least the general consistency between calculated and measured viscosities, to conclude that the viscous contribution reflects diffusion-accommodated grain-boundary sliding. This evidence of significant diffusional creep requiring grain-scale diffusion on timescales of 100 - 1000 s implies that much of the recoverable non-elastic strain evident in the microcreep records, and most of the dispersion and dissipation revealed by the forced oscillation tests at shorter periods, can plausibly be attributed to relaxation involving diffusional processes operative at scales smaller than the grain scale. Elastically accommodated (and thus reversible) migration and sliding of parts of grain-boundaries are prime candidates [Jackson et al., 1999]. Moreover, Gribb and Cooper [1998a] have demonstrated that the transient creep associated with the Raj and Ashby [1971] model for diffusion-accommodated grain-boundary sliding is well described by the Andrade creep function, with its implication of a continuous distribution of relaxation times.

There is also some evidence of the grainsize sensitivity expected of such phenomena in the results of Tan et al. [1997b; in prep.] where a marked reduction in the intensity of the viscoelastic relaxation accompanies grain growth from ~ 10 to ~ 150 μm in an initially fine-grained F090 specimen. Differences between the dissipation reported for relatively fine-grained synthetic olivine aggregates [Tan et al., 1997a, b, in prep.; Gribb and Cooper, 1998a] and the substantially lower values from previous studies of relatively coarse-grained natural ultramafic rocks [Berckhemer et al., 1982; Jackson et al., 1992] and olivine single-crystals [Gueguen et al., 1989] are also indicative of grainsize sensitivity. The data of Webb et al. [1999] and those of Tan et al. [in prep.] and Gribb and Cooper [1998c] indicate some enhancement of the 'high-temperature background' viscoelastic relaxation in materials containing relatively higher levels of impurity, expressed at high temperature by the presence of intergranular melt films.

*Extrapolation of Grainsize-Sensitive Behaviour to Mantle Grainsizes*

These observations are consistent with the dominance of grain-boundary processes in the relatively fine-grained materials, and work in progress may allow experimental quantification of the grainsize sensitivity of the coefficients β, η and n in the preferred Andrade model of the transient rheology. In the meantime, the theory of Raj and Ashby [1971] for diffusion-accommodated grain-boundary sliding provides a basis for an indicative extrapolation in grainsize within the diffusional regime [Gribb and Cooper, 1998a] as follows. The steady-state viscous strain rate εSS varies with grainsize as d^m with m = 2 for Nabarro-Herring (lattice diffusion) creep and m = 3 for Coble (grain-boundary diffusion) creep. The characteristic time τ for the relaxation by diffusional flow of the internal stresses caused by the grain-boundary sliding, is calculated as the ratio of the estimated amplitude of the transient strain (implicit in equation 23), which is grainsize-independent, to the steady-state strain rate εSS, yielding τ ~ d^n. The anelastic strain is a function only of the dimensionless time τ/ε. For the same value C of dimensionless time, the same amount of anelastic strain will have accumulated in materials of different grainsize. Applying this idea to the Andrade description of the transient strain, this condition becomes

\[ β(Cτ)^n - β(Cd_m)^n = f(d), \]  

so that the grainsize sensitivity required of β is

\[ β(d) ∼ d^{-mn} \]  

The grainsize-sensitive Andrade model thus becomes

\[ J(t,d) = J_U + β_0(d/d_0)^{mn} τ^n + t^{-1}(1/τ_0)(d/d_0)^{-m} \]  

where β_0 and (1/τ_0) are the coefficients determined at the reference value d_0 of the grainsize. This analysis, which applies to grainsize variation at fixed temperature, is based on the assumption that the exponent n in the transient term is not itself grainsize-sensitive.

Equation (28) provides for the extrapolation in grainsize of an experimentally determined Andrade rheology. The values of the frequency-dependent shear modulus G(ω) and of the strain-energy dissipation Q^{-1}(ω), associated with the grainsize-extrapolated rheology can then be calculated from the real and imaginary parts of the dynamic shear compliance (equations 9, 10 & 11).

This strategy for the extrapolation in grainsize has been applied to the preferred Andrade rheology derived from the 1300°C microcreep record 890-5 of Tan et al. (in prep.) for F090 olivine of 30 μm grainsize (Table 1). The calculated grainsize sensitivities of G(ω) and Q^{-1}(ω) are displayed in Fig. 13. The order-of-magnitude spacing of the dissipation curves (Fig. 13b) for grainsizes differing by a factor of 10 is consistent with control by the ~ d^{-1} grainsize dependence (here m = 3, n = 0.32) of the coefficient β of the transient term in the Andrade model, and is consistent with the previously mentioned general that the high-
temperature internal friction background in ceramic materials varies approximately as d^{-1} [Lakki and Schaller, 1996; Ota and Pezzoti, 1996]. The grain size sensitivity employed in this largely indicative extrapolation yields values of Q^{-1} < 0.003 for periods below 100 s at 3 mm grain size, significantly smaller than the levels of dissipation from seismological studies of the upper mantle (Q^{-1} > 0.01).

We have noted above that viscoelastic relaxation results in values of |∂G/∂T| at low frequencies and high temperatures which are much greater than the anharmonic derivative |∂G/∂T|_{lan} which applies at ultrasonic (MHz) frequencies (Fig. 9; see also [Karato, 1993]). Because the viscoelastic rheology is grain size-sensitive, so too is the magnitude of this enhancement of |∂G/∂T|. The extrapolation described above yields the grain size dependence of the modulus dispersion displayed in Fig. 13a. The dispersion becomes much less pronounced as the grain size increases. Average values of |∂G/∂T| for the temperature interval 900-1300°C can be estimated by comparing these frequency- and grain size-dependent values of G at 1300°C with the frequency- (and grain size)-independent value of 64.5 GPa at 900°C (Fig. 9). At 100 s period, the average value of |∂G/∂T| for the temperature interval 900-1300°C decreases from 6.5 × |∂G/∂T|_{lan} measured at 30 μm grain size to a calculated value of 3.7 × |∂G/∂T|_{lan} for 3 mm grain size. At 1 s period the corresponding change is from 4.5 to 3.7 times the anharmonic derivative. The convergence of the G(ω) trends for the various grain sizes towards the unrelaxed value (I_{U}^{-1}) of 44.6 GPa, rather than the value obtained by the ultrasonically-determined modulus of about 61 GPa (1300°C - Fig. 9), suggests that the unrelaxed compliance obtained from analysis of the creep record must substantially underestimate the true, infinite-frequency, asymptote (Tan et al., in prep.). It is thus evident that the grain size sensitivity of |∂G/∂T| will also have been underestimated.

Such modelling provides nothing more than a zeroth-order indication of the probable grain size sensitivity of Q^{-1} and G(ω) within the diffusional regime. It is clearly no substitute for direct experimental constraints on the grain size sensitivity of the transient rheology, some of which are likely to emerge from the data of Tan et al. (in prep.) for olivine aggregates of different grain sizes. It must also be stressed that it is distinctly possible that at grain sizes representative of the Earth's deep interior, and at the low stresses of teleseismic wave propagation, linear dislocation-migration mechanisms perhaps of the type discussed by Fantozzi et al. [1982] and Karato [1998] will dominate the transient creep and the associated phenomena of seismic wave dispersion and attenuation.

Viscoelastic Enhancement of |∂G/∂T|: a Complementary Approach

An alternative hybrid approach to the estimation of the viscoelastic enhancement of the temperature sensitivity of seismic wave speeds or moduli, that combines seismological determinations of Q^{-1} with emerging insights from the laboratory, was explored by Karato [1993]. He combined the expressions

\[ Q^{-1}(ω) \sim (ωτ_M)^{-α} \quad \text{and} \quad V(ω) \sim V_U [1 - (1/2) \cot (αω/2) Q^{-1}(ω)] \quad (29) \]

of Minster and Anderson [1981] describing the dissipation and phase speed (V) dispersion within the absorption band (i.e. for ωτ_m << 1 << ωτ_M) for the generalised Burgers
model in the low-loss limit \((Q^{-1} << 1)\). Specification of the temperature dependence of \(\tau_m\) through equation (20) above allowed differentiation to obtain an expression for \(\partial V/\partial T\), which, expressed in terms of the modulus \(G = \rho V^2\) is

\[
\partial \ln G/\partial T = \partial \ln G_I/\partial T - 2F(\alpha) [Q^{-1}(\omega/\pi)] (E/RT^2) \tag{30}
\]

In this equation, the first and second terms on the right-hand side are the anharmonic and viscoelastic contributions, respectively, and \(F(\alpha) = (\alpha \pi/2) \cot(\alpha \pi/2) - 1\) for \(0 < \alpha < 0.4\). Given experimental confirmation of the appropriateness of this absorption-band model, with power law exponents \(\alpha\) typically of 0.2 - 0.4 and preliminary indications of activation energies of 400 - 700 \(\text{kJ mol}^{-1}\), the anelastic enhancement of the temperature sensitivity of the wave speed or modulus in an anorthosite upper mantle can thus be estimated from seismological \(Q^{-1}\) data. In this way Karato demonstrated that viscoelastic behaviour associated with \(Q^{-1} \approx 0.01\) in the upper mantle would lead to an approximately two-fold enhancement of \(\partial \ln V_S/\partial T\) (or \(\partial \ln V_P/\partial T\)), and that similar effects are to be expected throughout the mantle. Allowance for viscoelastic enhancement of the temperature sensitivity of the shear wave speed results in smaller amplitudes for the thermal anomalies responsible for much of the lateral variability of \(V_S\), and also helps explain the relatively high values of \(\partial \ln V_S/\partial \ln V_P\), revealed by seismic tomography (e.g., Romanowicz and Durek, this volume).

**Dislocation Relaxation Mechanisms at Mantle Grainsizes**

It was inferred above that the diffusion-controlled grain-size-sensitive processes responsible for the viscoelastic behaviour of fine-grained synthetic ultramafic materials may contribute to, but probably do not dominate seismic wave attenuation in the upper mantle. Gribb and Cooper [1998a] speculated that this diffusion-controlled mechanism might be capable of producing the seismologically observed attenuation if sub-grain size rather than grain size were the controlling factor. It seems more likely that an alternative (dislocation) relaxation mechanism becomes dominant at larger grain sizes, resulting in higher values of dissipation than are inferred from this extrapolation of the influence of diffusion-accommodated grain-boundary sliding.

Much of the early work at high temperatures and seismic frequencies on relatively coarse-grained natural ultramafic rocks [Berckhemer et al., 1982; Jackson et al., 1992] and forsterite single-crystals [Guéguen et al., 1989] tends to indicate a role for dislocations. For example, broad consistency between the values of \(Q^{-1}\) measured on single crystals and rocks of \(\sim 0.5\) mm grain-size suggests minimal grain-size sensitivity at relatively large grain size. Furthermore, higher levels of dissipation measured on pre-deformed single-crystal forsterite than a undeformed equivalent, correlate with increased dislocation density in the pre-deformed specimen [Guéguen et al., 1989]. Markedly viscoelastic behaviour of single-crystal MgO similarly has been attributed to the stress-induced motion of dislocations [Getting et al., 1997]. However, the quantitative connection between the macroscopic viscoelastic behaviour of such relatively coarse-grained geological materials and the microdynamics of dislocation interaction with the applied stress field, sought for example by Minster and Anderson [1981], Karato and Spetzler [1990] and Karato [1998] remains to be demonstrated.

**The Effects of Elastic Anisotropy and (Inter-phase) Heterogeneity**

It has been argued above that anelastic relaxation of the internal stresses caused by the elastic anisotropy of the component crystallites is responsible for relaxation of the shear modulus from the Hashin-Shtrikman average \(G_{HS}\) to the Reuss lower bound. The relative modulus deficit \(\Delta G/G_{IJ} = (G_{HS} - G_{R})/G_{HS}\) probably needs to exceed 0.02 \((\Delta V_S/V_{SU} > 0.01)\) to be considered significant. The modulus relaxation is thus certainly significant in the case of bcc iron where \(\Delta G/G_{IJ}\) increases from 0.10 at 20°C to 0.39 at 900°C (Fig. 6), but is negligible for olivine where \(\Delta G/G_{IJ} < 0.025\) throughout the temperature interval 20-1200°C. For the MgSiO₃ perovskite phase thought to dominate the mineralogical constitution of the lower mantle, \(\Delta G/G_{IJ}\) is only 0.007 at 20°C [Teghaneh-Haeri, 1994], but its temperature dependence is unknown. For MgO, the elastic anisotropy increases markedly with increasing temperature causing \(\Delta G/G_{IJ}\) to increase from 0.021 at room temperature to 0.086 near 1500°C [Isaak et al., 1989]. Amongst these examples, degrees of elastic anisotropy sufficiently high to result in significant values of \(\Delta G/G_{IJ}\) are thus restricted to relatively simple cubic structures in which temperature typically has large differential effects on the two independent single-crystal shear moduli \(c_{44}\) and \(c' = (c_{11} - c_{12})/2\).

For seismological applications, the differential effect of pressure on the single-crystal shear moduli must also be considered. For the case of bcc iron, increasing pressure, like increasing temperature, increases the ratio \(c_{44}/c'\) [Guinan and Beshers, 1968], so that very substantial elastic anisotropy would be expected if this phase were stable under inner core conditions [Jackson et al., 1999]. For MgO, the differential effects of pressure and temperature are of opposite
sign [Jackson and Niesler, 1982; Isaak et al., 1989] and will tend to cancel.

Similar considerations must apply to multi-phase composites where the spacing between the calculated bounds on the effective moduli reflects differences between the directionally averaged properties of the constituent phases. Within the Earth's interior, these bounds are probably most widely spaced in the lower mantle, on account of the marked contrast in both bulk and shear modulus between the silicate perovskite and magnesiowüstite phases. For a model lower mantle of simplified pyrolite composition (comprising 81 vol % (Mg,Fe)SiO$_3$ perovskite and 19 vol % (Mg,Fe)O magnesiowüstite) at the zero-pressure foot of a putative 1600 K mantle adiabat [Jackson, 1998], the spacing $\Delta G/G_U$ between the Hashin-Shtrikman and Reuss averages is estimated to be about 0.03. This value is expected to decrease with increasing pressure along the adiabat because the pressure derivatives of the elastic moduli for the two phases are more similar than are their zero-pressure moduli.

It is concluded that the relaxation of internal stresses associated with elastic anisotropy and inter-phase heterogeneity, is likely to be observable in some laboratory situations such as bcc iron or MgO tested at high temperature, and could be seismologically very important especially if the bcc phase of iron were stable under inner core P-T conditions. Elsewhere, the relaxation of such internal stresses is unlikely to result in significant relaxation (> 1%) of the seismic shear wave speed.

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